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Keywords

design, controller, force, actuators, robust, elastic, series

Disciplines

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A Robust Force Controller Design for Series Elastic Actuators

Emre Sariyildiz, *Member, IEEE*, Haoyong Yu, *Member, IEEE*

Abstract— A Series Elastic Actuator (SEA) is designed by placing a passive compliant element between a conventional stiff actuator and link. The intrinsically compliant mechanical structure provides several superiorities, e.g., safety, energy efficiency, high force fidelity, low cost force measurement, high transparency, etc., in advanced robot applications, such as humanoids, quadrupeds and exoskeletons. However, the motion control problem of an SEA is more complicated than that of a conventional stiff actuator due to its higher order dynamics. This paper proposes a novel Active Disturbance Rejection (ADR) based robust force controller for SEAs by combining Differential Flatness (DF) and Disturbance Observer (DOb) in state space. The robust state and control input references are systematically generated in terms of a fictitious variable, namely differentially flat output, estimated disturbances and their successive derivatives. A second order DOB is designed in state space so that disturbances and their first and second order derivatives are estimated. It is experimentally shown that high performance force control applications can be performed without requiring the precise dynamic models of the actuator and environment when the proposed robust force controller is implemented.

I. INTRODUCTION

Stiff and non-back-drivable actuators intrinsically improve the performance of position control tasks thanks to their robust mechanical structures [1, 2]. However, they generally suffer from low performance, stability and safety problems in force control [3, 4]. High performance force control applications can be performed by using direct drive electromagnetic actuators [5]. Although they have relatively high power mass density, high power is available only at high speed with relatively low torque; i.e., they have low torque density which is a severe problem for many force control applications [6-8]. Speed reduction elements, such as gear, are generally used to improve the torque density of electromagnetic actuators; however, passive impedance, such as reflected inertia and damping, limits the bandwidth of force control and degrades the transparency in the transmission [6-9]. Moreover, nonlinear disturbances of drive mechanisms, such as friction and backlash, significantly influence the performance of force control [9]. Force sensors with feed-back algorithms are generally used

to compensate for the lack of transparency in the transmission and improve force fidelity; however, they have several shortcomings, such as stability problem, bandwidth limitation and noise, in practice [9-11]. Pneumatic and hydraulic actuators are considered to overcome the limitations of electromagnetic actuators, such as low torque density, and are used to perform different robot applications. However, they both suffer from nonlinearities which complicate the motion controller design; pneumatic actuators have low energy efficiency and constant need of pressurized air; and hydraulic actuators have high impedance due to friction and large fluid inertia [9, 12-14].

In recent years, Series Elastic Actuators (SEAs) have received increasing attention since they have several practical superiorities over conventional actuators in force control, e.g., lower mechanical output impedance, greater tolerance to impact load, lower stiction, higher force fidelity, and so on [9, 15, 16]. Therefore, they have been widely used in many advanced robot applications, e.g., industrial robots of the Rethink robotics, the Valkyrie and COMAN humanoid robots, and the RoboKnee and LOPES exoskeletons [17-21]. The motion control problem of SEAs is more complicated than that of conventional actuators due to their fourth order dynamic model. Conventionally, SEAs are controlled by using a single-loop PID force controllers, and the performance is improved by using Feed-Forward control [9]. However, it suffers from stability problem when nonlinear disturbances, such as friction and backlash, are significantly large [15]. The stability is improved by proposing a cascade control structure, i.e., designing a velocity controller in the inner-loop and impedance controller in the outer-loop [15, 22, 23]. However, the performance of the proposed controller is limited in real applications since the controller gains cannot be freely increased due to practical limitations, such as sampling time, noise and high frequency dynamics. To improve the robustness and performance of the real force control implementations, Disturbance Observer (DOb) was first applied to an SEA by Kong et al. [24]. The robust force controller was also similarly applied to the University of Texas's SEA (UT-SEA) and Valkyrie in [18, 25]. In these applications, DOB is designed for the velocity feed-back loop which is used to improve the stability of SEAs in [15, 22]. Since there are several parameters and higher order dynamics in the inner-loop, the design of DOB and tuning of nominal parameters are not straightforward. For example, one drawback of this design is that the motion control system may suffer from conservatism due to high order dynamics as explained in [26, 27]. Moreover, unexpected

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stability problem was reported in [28]. Authors have recently proposed a robust motion controller for SEAs by using resonance ratio control in [29]; however, it suffers from design complexity. A simple yet efficient robust motion controller design is still an open problem for SEAs.

In this paper, a novel ADR-based robust force controller, in which disturbances are directly treated by using their estimations, is proposed for SEAs by combining DF and DOB in state space. A simple yet efficient dynamic model of an SEA is obtained by using the analogy of a two-mass-spring-damper system. Its more complex dynamics, e.g., nonlinear friction, backlash, inertia variation, etc., are considered as internal disturbances in the design of the robust controller. Since precise dynamic model is not vital, the proposed robust force controller can be easily applied to many different SEAs in practice. In order to design a trajectory tracking controller in state space, the state and control input references are systematically generated in terms of a fictitious differentially flat output variable by using DF. The robustness of the motion controller, i.e., state feed-back controller, is achieved by modifying the state and control input references via the estimations of disturbances and their first and second order derivatives. They are obtained by using a second order DOB in state space. Force control goal of this paper is defined as the desired deflection of an SEA's spring. Therefore, it may be different from contact force. It is shown that the force control reference, i.e., the reference of the spring deflection, can be precisely followed when the proposed robust force controller is implemented. The validity of the proposal is verified by giving experimental results of an SEA.

The rest of the paper is organized as follows. In section II, the dynamic model of an SEA is given. In section III, the design of the second order DOB is briefly explained in state space. In section IV, a novel robust trajectory tracking controller is proposed by using DF and DOB. In section V, the robust controller is applied to the force control problem of SEAs. In section VI, the proposed robust force controller is experimentally verified. The paper ends with conclusion given in section VII.

II. SERIES ELASTIC ACTUATORS

Figure 1 illustrates the dynamic model of an SEA by using the analogy of a two-mass-spring-damper system. In this figure, m_i and b_i represent the i^{th} mass and viscous friction coefficient, respectively; q_i , \dot{q}_i , and \ddot{q}_i represent the position, velocity, and acceleration of the i^{th} mass, respectively; k_{12} represents the stiffness of the spring between the first and second masses; and F_{in} and F_{ext} represent input and external forces, i.e., motor torque and external load, respectively.

The dynamic equations of an SEA can be directly derived from Fig.1 as follows:

$$\begin{aligned} m_1 \ddot{q}_1 + b_1 \dot{q}_1 &= F_{in} - k_{12}(q_1 - q_2) - d_1 \\ m_2 \ddot{q}_2 + b_2 \dot{q}_2 &= k_{12}(q_1 - q_2) - d_2 \end{aligned} \quad (1)$$

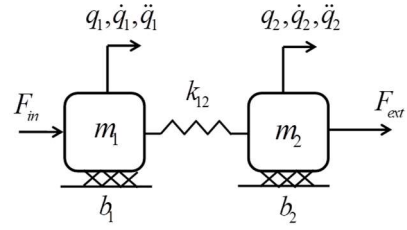


Fig. 1: Model of an SEA.

where m_n, b_n and k_{12n} represent the nominal parameters of m_i, b_i and k_{12} , respectively; and $d_1 = (m_1 - m_{1n})\ddot{q}_1 + (b_1 - b_{1n})\dot{q}_1 + (k_{12} - k_{12n})(q_1 - q_2) + f_{unm1}$ and $d_2 = (m_2 - m_{2n})\ddot{q}_2 + (b_2 - b_{2n})\dot{q}_2 + (k_{12n} - k_{12})(q_1 - q_2) + f_{unm2} + F_{ext}$ in which f_{unm1} and f_{unm2} represent any linear and nonlinear un-modeled disturbances, such as backlash and friction.

In this paper, the force control goal is defined by using $F_{spring}^{des} = k_{12}(q_1^{des} - q_2^{des})$ where \bullet^{des} represents desired \bullet . It can be considered as a position control goal, e.g. $q_1^{des} - q_2^{des} = F_{spring}^{des} / k_{12}$. It is one of the fundamental superiorities of SEAs over conventional stiff actuators in force control.

Eq. (1) can be represented in state space as follows:

$$\dot{x} = A_n x + B_n u - \tau_{dis} \quad (2)$$

$$\text{where } A_n = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{k_{12n}}{m_{1n}} & \frac{b_{1n}}{m_{1n}} & \frac{k_{12n}}{m_{1n}} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k_{12n}}{m_{2n}} & 0 & \frac{k_{12n}}{m_{2n}} & \frac{b_{2n}}{m_{2n}} \end{bmatrix}, B_n = \begin{bmatrix} 0 \\ \frac{1}{m_{1n}} \\ 0 \\ 0 \end{bmatrix}, \tau_{dis} = \begin{bmatrix} 0 \\ \frac{d_1}{m_{1n}} \\ 0 \\ \frac{d_2}{m_{2n}} \end{bmatrix},$$

$$x = [q_1 \quad \dot{q}_1 \quad q_2 \quad \dot{q}_2], u = F_{in}.$$

It can be easily verified by using Eq. (2) and the controllability matrix, $\Gamma = [B_n \quad A_n B_n \quad A_n^2 B_n \quad A_n^3 B_n]$, that all states of an SEA are controllable. As it is shown in section IV, the controllability is the necessary and sufficient condition to design the proposed robust force controller.

Equation (2) shows that collocated and non-collocated disturbances influence the dynamic model of an SEA. The collocated disturbance, d_1 , acts system via the second channel in which there is control input. Therefore, it can be easily cancelled by feeding-back its estimation. However, there is no control input in the channel of the non-collocated disturbance, d_2 . Therefore, it cannot be suppressed/cancelled by using conventional ADR control methods [29].

III. DISTURBANCE OBSERVER

As shown in the next section, disturbances and their first and second order derivatives are required in the design of the proposed robust motion controller. Therefore, the second order DOB is briefly explained in this section. It is designed by assuming that the third order derivatives of disturbances are zero, i.e., $\ddot{\tau}_{dis} \cong 0$. Similar assumptions are widely used in the design of DOB. Reader is invited to refer to [30] for further details on DOB-based robust motion control systems.

Let us first define auxiliary variables in terms of the disturbance and state vectors, which are given in Eq. (2), by using

$$\begin{aligned} z_1 &= \tau_{dis} + L_1 x \\ z_2 &= \dot{\tau}_{dis} + L_2 x \\ z_3 &= \ddot{\tau}_{dis} + L_3 x \end{aligned} \quad (3)$$

where $z_i \in R^4$ represents the i^{th} auxiliary variable vector; $L_i \in R$ represents the i^{th} gain of DOB; and $\dot{\tau}_{dis}$ and $\ddot{\tau}_{dis} \in R^4$ represent the first and second order time derivatives of the disturbance vector, i.e., τ_{dis} .

The time derivatives of auxiliary variables are derived as follows:

$$\begin{aligned} \dot{z}_1 &= -L_1 z_1 + z_2 + L_1 (A_n x + b_n u + L_1 x) - L_2 x \\ \dot{z}_2 &= -L_2 z_1 + z_3 + L_2 (A_n x + b_n u + L_1 x) - L_3 x \\ \dot{z}_3 &= -L_3 z_1 + L_3 (A_n x + b_n u + L_1 x) \end{aligned} \quad (4)$$

where $\dot{z}_i \in R^4$ represents the time derivative of the i^{th} auxiliary variable vector, z_i .

Equation (4) is derived in terms of nominal plant parameters. Since control input is known and system states are measured, the estimations of the auxiliary variables can be simply obtained by substituting them into Eq. (4) as follows:

$$\begin{aligned} \dot{\hat{z}}_1 &= -L_1 \hat{z}_1 + \hat{z}_2 + L_1 (A_n x + b_n u + L_1 x) - L_2 x \\ \dot{\hat{z}}_2 &= -L_2 \hat{z}_1 + \hat{z}_3 + L_2 (A_n x + b_n u + L_1 x) - L_3 x \\ \dot{\hat{z}}_3 &= -L_3 \hat{z}_1 + L_3 (A_n x + b_n u + L_1 x) \end{aligned} \quad (5)$$

where $\hat{z}_i \in R^4$ represents the estimation of the i^{th} auxiliary variable, and $\dot{\hat{z}}_i$ represents the time derivative of \hat{z}_i .

If Eq. (5) is subtracted from Eq. (4) and $e_i = z_i - \hat{z}_i$, which represents the estimation error of the i^{th} auxiliary variable, is substituted, then the dynamic equation of the auxiliary variable estimation error is derived in matrix form as follows:

$$\dot{e} = \Psi e \quad (6)$$

where $e = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}$; $\Psi = \begin{bmatrix} -L_1 I_4 & I_4 & 0_4 \\ -L_2 I_4 & 0_4 & I_4 \\ -L_3 I_4 & 0_4 & 0_4 \end{bmatrix} \in R^{12 \times 12}$ represents

the characteristic matrix of the auxiliary variable estimation error; and I_4 and 0_4 represent 4×4 identity and null matrices, respectively.

The bandwidth of DOB is directly related to the eigenvalues of Ψ and can be tuned by adjusting the gains of DOB as follows:

$$\begin{aligned} \det(\lambda I_{12} - \Psi) &= (\lambda^3 + L_1 \lambda^2 + L_2 \lambda + L_3)^4 = 0 \\ (\lambda + g_{DOB})^3 &= \lambda^3 + L_1 \lambda^2 + L_2 \lambda + L_3 \end{aligned} \quad (7)$$

where λ represents the eigenvalue of Ψ , and g_{DOB} represents the bandwidth of DOB.

If all eigenvalues of Ψ are negative, then $e \rightarrow 0$ or $\hat{z}_i \rightarrow z_i$ asymptotically. Hence, the estimations of the disturbance vector and its first and second order derivatives are derived as follows:

$$\begin{aligned} \hat{\tau}_{dis} &= \hat{z}_1 - L_1 x \\ \dot{\hat{\tau}}_{dis} &= \hat{z}_2 - L_2 x \\ \ddot{\hat{\tau}}_{dis} &= \hat{z}_3 - L_3 x \end{aligned} \quad (8)$$

where $\hat{\tau}_{dis}$, $\dot{\hat{\tau}}_{dis}$ and $\ddot{\hat{\tau}}_{dis} \in R^4$ represent the estimations of τ_{dis} , $\dot{\tau}_{dis}$ and $\ddot{\tau}_{dis}$, respectively.

Larger magnitude eigenvalues correspond to higher bandwidth of DOB and faster estimations of auxiliary variables. However, they are limited by practical constraints such as noise of measurement and sampling time.

IV. ROBUST TRAJECTORY TRACKING CONTROLLER DESIGN IN STATE SPACE

In this section, a novel robust trajectory tracking controller is proposed by combining DF and DOB. Reader, who is only interested in the practical applications of SEAs, can skip it and follow the next section.

A system is differentially flat if all state variables and control inputs are expressed in terms of a set of fictitious independent variables called differentially flat output and a finite number of its successive time derivatives [31].

Let us consider a general nonlinear dynamic model for a differentially flat system by using

$$\dot{x} = f(x, u) \quad (9)$$

where $x \in R^n$ represents system states and $u \in R^m$ represents control input. A fictitious differentially flat output variable is defined for such system by using

$$y_{DFO} = \varphi \left(x, u, \dot{u}, \dots, u^{(l)} \right) \quad (10)$$

where $y_{DFO} \in R^m$ and l is a finite m-tuple of integers, such that

$$\begin{aligned} x &= \kappa_x \left(y_{DFO}, \dot{y}_{DFO}, \ddot{y}_{DFO}, \dots, y_{DFO}^{(r)} \right) \\ u &= \kappa_u \left(y_{DFO}, \dot{y}_{DFO}, \ddot{y}_{DFO}, \dots, y_{DFO}^{(r+1)} \right) \end{aligned} \quad (11)$$

where r is a finite m-tuple of integers [32].

A linear system is flat if and only if it is controllable [32]. Equation (11) can be rewritten for linear systems as follows:

$$\begin{aligned} x_i &= \sum_{j=1}^m \sum_{k=1}^{q_j} \alpha_{ijk}^{(k)} y_{DFOj}, \quad i=1, 2, \dots, n \\ u_i &= \sum_{j=1}^m \sum_{k=1}^{q_j+1} \beta_{ijk}^{(k)} y_{DFOj}, \quad i=1, 2, \dots, m \end{aligned} \quad (12)$$

The conventional DF-based trajectory tracking controller is impractical for many motion control applications since plant uncertainties and external disturbances are not considered in its design [32, 33]. The following theorem proposes a novel DF-based robust trajectory tracking

controller in state space. It can be designed by using the estimations of disturbances in real implementations.

Theorem: Let us describe the dynamic model of a linear time-invariant and controllable system in polynomial matrix form by using

$$A_n(s)x(s) = B_n(s)u - D(s) \quad (13)$$

where $A_n(s) \in R^{n \times n}$ represents system matrix; $B_n(s) \in R^{n \times m}$ represents control input matrix; $x(s) \in R^n$ represents system states; $u \in R^m$ represents control input; $D(s) \in R^n$ represents disturbance vector; and s represents complex Laplace variable.

The robust state and control input references can be generated by using

$$x^{ref}(s) = \tilde{P}(s)y_{DFO} = P_1(s)y_{DFO} + P_2(s)D(s) \quad (14)$$

$$u^{ref} = \tilde{Q}(s)y_{DFO} = Q_1(s)y_{DFO} + Q_2(s)D(s) \quad (15)$$

where $\tilde{P}(s)y_{DFO}$ is derived by solving

$$C^T A_n(s) \tilde{P}(s)y_{DFO} + C^T D(s) = 0 \quad (16)$$

in which C^T is orthogonal to $B_n(s)$, i.e., $C^T B_n(s) = 0$; $Q_1(s)$ and $Q_2(s)$ are obtained by using

$$Q_1(s) = (B_n^T(s)B_n(s))^{-1} B_n^T(s)A_n(s)P_1(s) \quad (17)$$

$$Q_2(s) = (B_n^T(s)B_n(s))^{-1} B_n^T(s)(A_n(s)P_2(s) + I) \quad \square$$

Proof: Since the linear time-invariant system is controllable, states and control input can be defined in terms of differentially flat output variable. Eq. (13) can be rewritten by using

$$A_n(s)\tilde{P}(s)y_{DFO} + R(s)y_{DFO} = B_n(s)\tilde{Q}(s)y_{DFO} \quad (18)$$

$$A_n(s)\tilde{P}(s) + R(s) = B_n(s)\tilde{Q}(s) \quad (19)$$

where $x(s) = \tilde{P}(s)y_{DFO}$, $u = \tilde{Q}(s)y_{DFO}$ and $D(s) = R(s)y_{DFO}$ in which $R(s) = B_n(s)\tilde{Q}(s) - A_n(s)\tilde{P}(s)$.

If Eq. (18) is multiplied by C^T , which is orthogonal to $B_n(s)$, from the left side, then Eq. (16) is derived as follows:

$$C^T A_n(s) (\tilde{P}(s) + A_n^{-1}(s)R(s)) = 0 \quad (20)$$

Let us first prove the existence of the solution of Eq. (20). The Smith form of the matrix $C^T A_n(s) \in R^{m \times n}$ $m \leq n$ can be derived as follows:

$$VC^T A_n(s)U = VC^T A_n(s)[U_1 | U_2] = [\Delta | O] \quad (21)$$

where $U \in R^{n \times n}$ and $V \in R^{m \times m}$ are two unimodular matrices in which $U_1 \in R^{m \times n-m}$ and $U_2 \in R^{m \times m}$; $\Delta \in R^{n-m \times n-m}$ is a diagonal matrix; and $0 \in R^{n-m \times m}$ is a null matrix [32].

Let us assume that $S \in R^{m \times m}$ is an arbitrary unimodular matrix that satisfies

$$\tilde{P}(s) + A_n^{-1}(s)R(s) = U \begin{bmatrix} 0 \\ S \end{bmatrix} = U_2 S \quad (22)$$

where $0 \in R^{n-m \times m}$ is a null matrix.

If Eq. (22) is applied into Eq. (21), then

$$VC^T A_n(s) (\tilde{P}(s) + A_n^{-1}(s)R(s)) = VC^T A_n(s)U_2 S = 0 \quad (23)$$

Equation (23) shows that Eq. (22) is the solution of Eq. (20). Hence, $\tilde{P}(s)$ is derived by using

$$\begin{aligned} \tilde{P}(s) &= U_2 S - A_n^{-1}(s)R(s) \\ \tilde{P}(s)y_{DFO} &= U_2 S y_{DFO} - A_n^{-1}(s)R(s)y_{DFO} = P_1(s)y_{DFO} + P_2(s)D(s) \end{aligned} \quad (24)$$

If Eq. (24) is applied into Eq. (18), then

$$A_n(s)P_1(s)y_{DFO} + A_n(s)P_2(s)D(s) + D(s) = B_n(s)\tilde{Q}(s)y_{DFO} \quad (25)$$

Equation (17) can be directly derived by multiplying Eq. (25) with $(B_n^T(s)B_n(s))^{-1} B_n^T(s)$ from the left side. \square

V. ROBUST FORCE CONTROL OF AN SEA

In this section, a novel ADR controller is proposed for the robust force control problem of SEAs by using **Theorem**.

The dynamic model of an SEA, which is given in Eq. (1), can be described in polynomial matrix form by using

$$A_n(s)x(s) + D(s) = B_n(s)u \quad (26)$$

where $A_n(s) = \begin{bmatrix} m_n s^2 + b_n s + k_{12n} & -k_{12n} \\ -k_{12n} & m_{2n} s^2 + b_{2n} s + k_{12n} \end{bmatrix}$; $x(s) = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$;

$B_n(s) = [1 \ 0]^T$; $D(s) = \begin{bmatrix} \hat{d}_1 & \hat{d}_2 \end{bmatrix}^T$; $u = F_m$; s represents complex Laplace variable; and \hat{d}_1 and \hat{d}_2 represent the estimations of d_1 and d_2 , respectively. In order to design the robust force controller, the estimations of disturbances are used instead of real disturbances.

Equation (16) is derived by multiplying Eq. (26) with $C^T(s) = [0 \ 1]$ from left side. The robust state references are derived from Eq. (14) as follows:

$$x^{ref} = \tilde{P}(s)y_{DFO} = \begin{bmatrix} m_{2n}\ddot{y}_{DFO} + b_{2n}\dot{y}_{DFO} + k_{12n}y_{DFO} + \hat{d}_2/k_{12n} \\ k_{12n}y_{DFO} \end{bmatrix} \quad (27)$$

The robust control input is derived from Eq. (15) as follows:

$$\begin{aligned} u^{ref} &= m_{1n}m_{2n}\ddot{y}_{DFO} + (m_{1n}b_{2n} + m_{2n}b_{1n})\dot{y}_{DFO} + (b_{1n}b_{2n} + k_{12n}(m_{1n} + m_{2n}))y_{DFO} + \\ &k_{12n}(b_{1n} + b_{2n})\dot{y}_{DFO} + \hat{d}_1 + \hat{d}_2 + \frac{b_{1n}}{k_{12n}}\hat{d}_2 + \frac{m_{1n}}{k_{12n}}\hat{d}_2 \end{aligned} \quad (28)$$

The robust state and control input references are derived in terms of differentially flat output variable, estimated disturbances, and their successive time derivatives; however, the desired differentially flat output variable has yet to be determined. It can be derived by using the force control goal as follows:

$$k_{12n}(p_1(s) - p_2(s))y_{DFO}^{des} = k_{12n}(m_{2n}\ddot{y}_{DFO}^{des} + b_{2n}\dot{y}_{DFO}^{des}) + \hat{d}_2 = F_{spring}^{des} \quad (29)$$

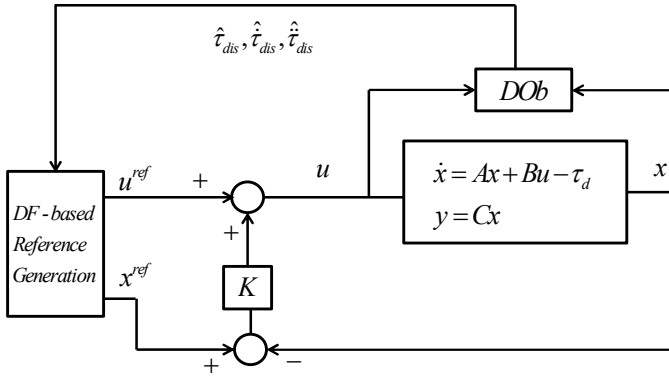


Fig. 2: Block diagram of the proposed robust force control system.

$$\ddot{y}_{DFO}^{des} = \frac{F_{spring}^{des} - \hat{d}_2 - b_{2n}k_{12n}\dot{y}_{DFO}^{des}}{k_{12n}m_{2n}} \quad (30)$$

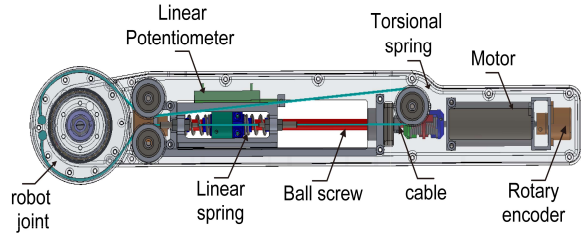
where y_{DFO}^{des} represents the desired differentially flat output variable.

The block diagram of the proposed robust force control system is shown in Fig. 2. In this figure, K represents the conventional state feed-back control gain. It can be tuned by using conventional pole placement method for the nominal plant model.

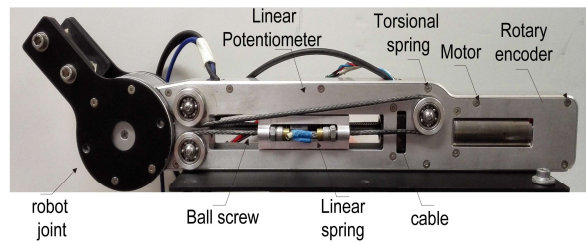
VI. EXPERIMENTS

In this section, force control experimental results of an SEA, which is shown in Fig. 3, are given. It has a novel mechanical structure which consists of torsional and linear springs in series. A compact variable stiffness SEA is simply achieved by adjusting the compliance of the springs; e.g., a hard torsional spring and a soft linear spring are used in our design. Figure 3a and Fig. 3b illustrate the composition and the second prototype of the novel actuator, respectively. The reader is invited to refer to [28] for further details on the novel actuator design and control. However, in this paper, only the torsional spring is used as a conventional SEA to validate the proposed robust force controller.

Specifications of the experimental setup are shown in



a) Principle of the actuator design (CAD model)



b) A prototype of the novel series elastic actuator.

Fig. 3: Series Elastic Actuator.

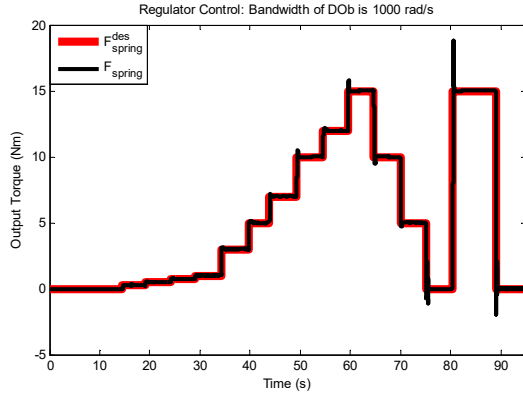
TABLE I. SPECIFICATIONS OF EXPERIMENTAL SETUP

Parameters	Description	Values
m_1	Inertia of motor	$2.2 \times 10^{-6} \text{ kgm}^2$
m_2	Inertia of link	$4 \times 10^{-6} \text{ kgm}^2$
k_{12}	Spring stiffness	0.14 Nm/rad

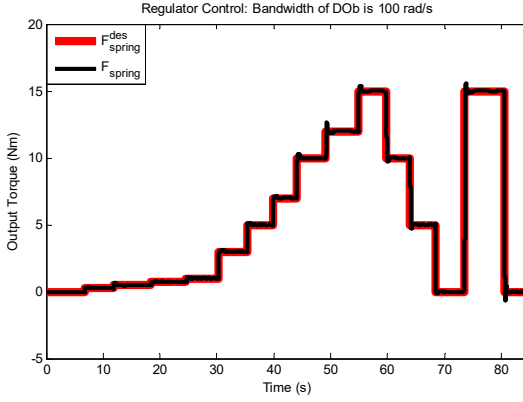
Table I. The center processor is dSPACE DS1007 with DS3002 counter board to collect encoder signals and DS2102 DAC board to control motor driver. The motor is Maxon EC-4pole-30 and the motor driver is Elmo Harmonica 12/60. The sampling frequency of the experiments is 2 KHz. Regulation and trajectory tracking control experiments are performed to validate the controller.

Firstly, let us consider the force regulation control problem. The state feed-back controller is designed for the nominal plant model by neglecting disturbances. In order to assign the double poles of the nominal system at -75 and -100, the state feed-back controller is designed by using $K = [-0.1345 \ 0.0006 \ 0.1380 \ -0.0006]$. Step reference inputs are consecutively applied by using 0.25Nm, 0.5Nm, 0.75Nm, 1Nm, 3Nm, 5Nm, 7Nm, 10Nm, 13Nm, 15Nm, 10Nm, 5Nm, 0Nm, 15Nm and 0Nm. In order to show the contact stability and performance for different environmental dynamics, e.g. stiffness, a sponge is placed between the actuator link and stiff environment that is metal. As the force control input is increased, the dynamics of stiff environment becomes more dominant. Figure 4a and Fig. 4b show that the proposed robust force controller can accurately follow the step reference inputs for different bandwidth values of DOB. At low force reference range, the actuator contacts to soft environment, i.e., sponge. However, as the force reference input is increased, it starts to contact to hard environment, i.e., metal. The proposed robust force controller can satisfy stable and high-performance contact motion for different environmental dynamics. Although the robustness deteriorates as the bandwidth of DOB is decreased, high performance contact motion can still be achieved. Figure 4c shows the estimation of the disturbance in the fourth channel, i.e., the disturbance at link side. It is directly related to the contact motion in force control; i.e., as the force reference input is increased the estimated disturbance increases as well. It is a well-known fact that the accuracy of disturbance estimation improves as the bandwidth of DOB is increased. However, the estimation suffers from noise as shown in Fig. 4c. The trade-off between the accuracy of disturbance estimation and noise-sensitivity should be kept in mind in the design of the proposed robust force controller.

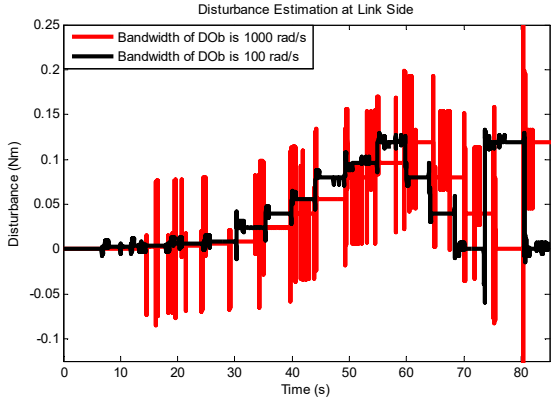
Let us now consider the force trajectory tracking control problem. The state feed-back controller is similarly designed as $K = [-0.0445 \ 0.0009 \ 0.0587 \ -0.0006]$ so that the double poles of the nominal system are placed at -120 and -125. The trajectory reference input is applied by using $F_{spring}^{des} = 7 + 3\sin(\omega t) \text{ Nm}$. Figure 5a shows the robust force control result when the frequency of the reference input is 1 Hz. As it is shown in the figure, high performance robust force control can be performed when the bandwidth of DOB



a) Force control results when DOB's bandwidth is 1000 rad/s.



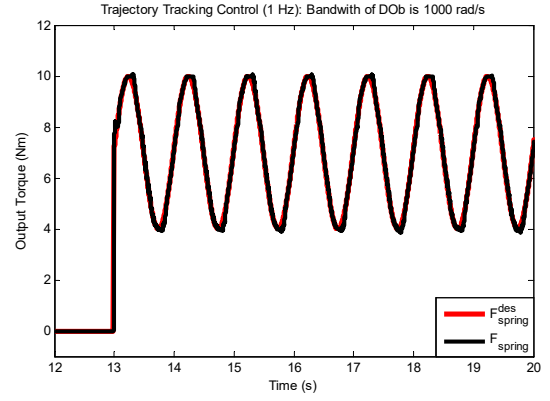
b) Force control results when DOB's bandwidth is 100 rad/s.



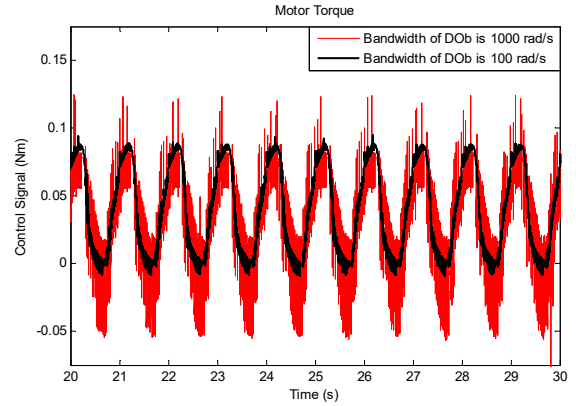
c) Estimations of disturbances in the fourth channel when DOB is designed by using different bandwidth values.

Fig. 4: Regulation control results.

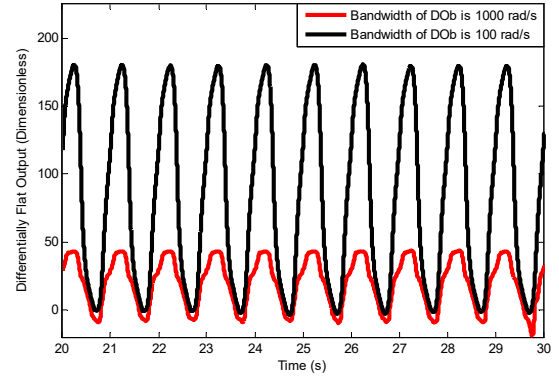
is 1000 rad/s. The performance of force control deteriorates as its bandwidth is decreased. The control signals are shown in Fig. 5b when the bandwidths of DOB are set as 100 rad/s and 1000 rad/s. It is clear from the figure that although the robustness of force control is improved by increasing the bandwidth of DOB, it becomes more noise-sensitive. The bandwidth of DOB should be experimentally tuned by considering the trade-off between the robustness and noise-sensitivity. Figure 5c shows that as the bandwidth of DOB is increased, lower differentially flat output variable is obtained. Figure 5d shows the robust force control results when the frequency of the reference input is 5 Hz. It is clear from the figure that force control can be precisely performed for high frequency reference inputs when the proposed robust controller is implemented.



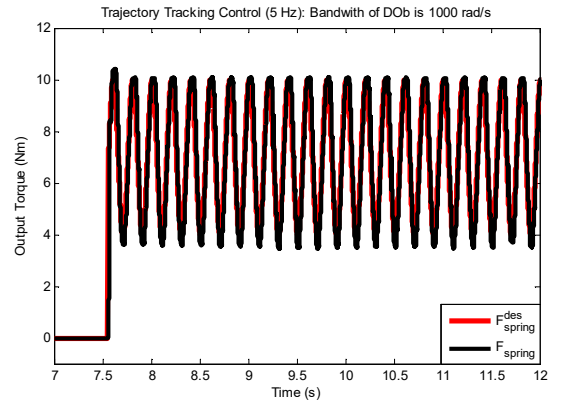
a) Force control results (1 Hz).



b) Control signal.



c) Differentially flat output variable.



d) Force control result (5 Hz).

Fig. 5: Trajectory tracking control results.

VII. CONCLUSION

This paper has proposed a novel ADR-based robust force controller for SEAs by using DF and the second order DOB in state space. It is experimentally verified that high performance force control applications can be performed without requiring the exact dynamic models of the actuator and environment. Active force control experiments have been performed by keeping contact stability when SEA interacts with different environments, i.e., sponge and metal. In order to apply the proposed robust force controller to advanced robot applications, such as rehabilitation and assistive robotics, the stability should be further investigated by considering dynamic and active environments such as human beings. The stability and performance of the proposed controller can be improved by adaptively tuning the nominal design parameters and state feed-back controller gain.

The proposed robust motion controller has a two-degrees-of-freedom control structure. Its robustness and performance can be independently adjusted by tuning DOB and state feed-back controller, respectively. The robustness and performance of the proposed controller is limited by practical constraints such as noise and sampling time. Therefore, one should consider the practical constraints in the design of the proposed robust motion controller.

Although only the force control problem of SEAs is considered in this paper, the proposed controller can be similarly applied to their position control problem by only modifying the desired differentially flat output variable. Therefore, the proposed controller is very practical for different motion control applications of compliant robotic systems.

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